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Optimal Scheduling of Fuel-Minimal Approach Trajectories

Florian Fisch, Matthias Bittner, Prof. Florian Holzapfel
Institute of Flight System Dynamics, Technische Universität München, Garching, Germany
Outline

1. Introduction
2. Aircraft Simulation Model
3. Multi-Aircraft Optimization Problem
4. Results
5. Summary & Outlook
1. Introduction

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Introduction

Computation of fuel minimal and noise minimal approach trajectories:

So far:
⇒ Optimization of *stand-alone* approach trajectories
⇒ Limitations due to other aircraft in the vicinity of an airport are not taken into account
⇒ Optimization results can not be put into practice due to the limitations arising from the remaining air traffic and the daily airport business

Here:
⇒ *Simultaneous* optimization of the approach trajectories of *multiple* aircraft present in the vicinity of an airport
⇒ Landing sequence is not pre-determined and has to be found by the optimization procedure
⇒ More realistic results
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Aircraft Simulation Model

Point-Mass Simulation Model:

Position Equations of Motion (\textit{NED}-Frame):

\[
\begin{pmatrix}
\dot{x}^E \\
\dot{y} \\
\dot{z}_O
\end{pmatrix}
= \begin{pmatrix}
V_K^G \cdot \cos \chi_K^G \cdot \cos \gamma_K^G \\
V_K^G \cdot \sin \chi_K^G \cdot \cos \gamma_K^G \\
-V_K^G \cdot \sin \gamma_K^G
\end{pmatrix}_O
\]

Translation Equations of Motion:
Aircraft Simulation Model

Total sum of external forces:

\[
\left( \sum F^G \right)_K = \left( F_A^G \right)_K + \left( F_P^G \right)_K + M_{KO} \cdot \left( F_G^G \right)_O
\]

Thrust modeling:

\[
T = \delta_T \cdot T_{\text{max}}
\]

\[
T_{\text{max}} = T_{\text{max,ISA}} \cdot \left( 1 - C_{Tc5} \cdot \Delta T_{ISA,\text{eff}} \right)
\]

\[
T_{\text{max,ISA}} = C_{Tc1} \cdot \left( 1 - \frac{h}{C_{Tc2}} + C_{Tc3} \cdot (h)^2 \right)
\]

\[
\Delta T_{ISA,\text{eff}} = \Delta T_{ISA} - C_{Tc4}
\]

Aerodynamic coefficients:

\[
C_D = C_{D0} + C_{D2} \cdot C_L^2
\]

\[
C_L = C_{L0} + C_{La} \cdot \alpha_{A,\text{CMD}}
\]
Aircraft Simulation Model

Aerodynamic Forces:

\[ D = \bar{q} \cdot S \cdot C_D = \bar{q} \cdot S \cdot \left( C_{D0} + C_{D2} \cdot C_L^2 \right) \]

\[ Q = \bar{q} \cdot S \cdot C_Q = \bar{q} \cdot S \cdot C_{Q\beta} \cdot \beta_A = 0 \]

\[ L = \bar{q} \cdot S \cdot C_L = \bar{q} \cdot S \cdot \left( C_{L0} + C_{La} \cdot \alpha_A \right) \]

Dynamic pressure:

\[ \bar{q} = 0.5 \cdot \rho \cdot V_A^2 \]

Force vector:

\[ \left( \bar{F}^G \right)_A = \left( \bar{F}^G \right)_A + \left( \bar{F}^G \right)_A = \begin{pmatrix} -D \\ 0 \\ -L \end{pmatrix} + \begin{pmatrix} T \\ 0 \end{pmatrix} = \begin{pmatrix} T - D \\ 0 \\ -L \end{pmatrix} \]
Aircraft Simulation Model

Noise model:

Sound pressure level:

\[ L_A(T, r) = a \cdot T + b \cdot \log(r) + c \cdot \log(r)^2 + d \]

Sound exposure level:

\[ L_{AE} = 10 \log\left( \frac{1}{t_0} \int_{t_1}^{t_2} 10^{L_A(t)/10} \, dt \right) \]

Number ofAwakenings:

\[ n_{AW} = \sum_i 0.0087 \cdot (L_{AE,i} - 30)^{1.79} p_i \]
Aircraft Simulation Model

Atmospheric model (DIN ISO 2533):

\[ H_G = \frac{r_E \cdot h}{r_E + h} \]

\[ \rho = \rho_S \left[ 1 + \frac{\gamma_T r}{T_S} \cdot H_G \right] \left( -\frac{g_S}{R \gamma_T} \cdot 1 \right) \]

\[ p = p_S \left[ 1 + \frac{\gamma_T r}{T_S} \cdot H_G \right] \left( -\frac{g_S}{R \gamma_T} \right) \]

Fuel consumption:

\[ \dot{m}_{\text{fuel, idle}} = C_{f3} \cdot \left( 1 - \frac{h}{C_{f4}} \right) \]

\[ \dot{m}_{\text{fuel, max}} = C_{f1} \cdot \left( 1 + \frac{V_A}{C_{f2}} \right) \cdot T_{\text{max}} \]

\[ \dot{m}_{\text{fuel}} = \dot{m}_{\text{fuel, idle}} + \delta_T \cdot (\dot{m}_{\text{fuel, max}} - \dot{m}_{\text{fuel, idle}}) \]
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Multi-Aircraft Optimization Problem

Determine the optimal control histories
\[ u_{i,\text{opt}}(t_i) \in P^m \]
and the corresponding optimal state trajectories
\[ x_{i,\text{opt}}(t_i) \in P^n \]
that minimize the Bolza cost functional
\[ J = \sum_{i=1}^{N} \left[ e_i(x_i(t_f), t_{f,i}) + \int_{t_{0,i}}^{t_{f,i}} L_i(x_i(t), u_i(t), t_i) dt_i \right] \]
subject to

⇒ the state dynamics
\[ \dot{x}_i(t_i) = f_i(x_i(t_i), u_i(t_i), t_i) \]
⇒ the initial boundary conditions
\[ \psi_{0,i}(x_i(t_{0,i}), t_{0,i}) = 0 \quad \psi_{0,i} \in P^{q_i} \]
⇒ the final boundary conditions
\[ \psi_{f,i}(x_i(t_{f,i}), t_{f,i}) = 0 \quad \psi_{f,i} \in P^{p_i} \]
⇒ the interior point conditions
\[ r_i(x(t_i), t_i) = 0 \quad r_i \in P^{k_i} \]
⇒ the equality constraints
\[ C_{eq,i}(x_i(t_i), u_i(t_i), t_i) = 0 \quad C_{eq,i} \in P^{r_i} \]
⇒ and the inequality constraints
\[ C_{ineq,i}(x_i(t_i), u_i(t_i), t_i) \leq 0 \quad C_{ineq,i} \in P^{s_i} \]
\[ i = 1, \ldots, N \]
Initial boundary conditions:

- Defined by the entry position into the considered air space

Final boundary conditions:

- Assure that the aircraft are finally located on the ILS glide path
- Final approach fix: located at the origin of the Local Fixed Frame $N$ at an altitude of $h_{FAF}$
- ILS glide path: directed parallel to the $x$-axis of the Local Fixed Frame $N$, into the direction of the positive $x$-axis
Multi-Aircraft Optimization Problem

Final boundary conditions:

- **Northward position:** \( x(t_f) \geq x_{FAF} + \Delta x \)
- **Eastward position:** \( y(t_f) = y_{FAF} \)
- **Altitude:** \( h(t_f) = h_{FAF} + \tan(-\gamma_{K,ILS}) \cdot x(t_f) \)
- **Glide-path angle:** \( \gamma_K(t_f) = \gamma_{K,ILS} \)
- **Heading angle:** \( \chi_K(t_f) = \chi_{K,ILS} \)
- **Kinematic velocity:** \( V_K(t_f) = V_{K,ILS} \)
Multi-Aircraft Optimization Problem

Inequality path constraints:

⇒ Load factor: \( n_{Z, LB} = 0.85 \leq n_Z(t) \leq 1.15 = n_{Z, UB} \)

⇒ Kinematic velocity: \( V_{K, LB} = 200 \frac{km}{h} \leq V_K(t) \leq 1000 \frac{km}{h} = V_{K, UB} \)

⇒ Angle of attack: \( \alpha_{A, CMD, LB} = -5.73^\circ \leq \alpha_{A, CMD} \leq 20.05^\circ = \alpha_{A, CMD, UB} \)

⇒ Bank angle: \( \mu_{K, CMD, LB} = -45^\circ \leq \mu_{K, CMD} \leq 45^\circ = \mu_{K, CMD, UB} \)

⇒ Thrust lever: \( \delta_{T, CMD, LB} = 0.0 \leq \delta_{T, CMD} \leq 1.0 = \delta_{T, CMD, UB} \)

⇒ Eastward position: \( y_{LB}(t) \leq y(t) \leq y_{UB}(t) \)

⇒ Altitude: \( h_{LB}(t) \leq h(t) \leq h_{UB}(t) \)

⇒ Aircraft distance: \( d_{ij}(t) - d_{\text{min}} \geq 0 \)
Inequality path constraints:

⇒ Path constraints are formulated such that the aircraft have to follow the ILS glide path once the FAF has been passed.

⇒ Eastward position:

\[
y_{LB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot y_{LB} - 100.0
\]
\[
y_{UB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot y_{UB} + 100.0
\]

⇒ Altitude:

\[
h_{LB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot h_{LB} + (h_{FAF} - 100.0) + \tan(-\gamma_{ILS}) \cdot x(t)
\]
\[
h_{UB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot h_{UB} + (h_{FAF} + 100.0) + \tan(-\gamma_{ILS}) \cdot x(t)
\]

⇒ Kinematic velocity:

\[
V_{K, LB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot (V_{K, LB} - V_{K, ILS} + 10.0) + (V_{K, ILS} - 10.0)
\]
\[
V_{K, UB}(t) = 0.5 \cdot [1 - \tanh(a \cdot x(t))] \cdot (V_{K, UB} - V_{K, ILS} - 10.0) + (V_{K, ILS} + 10.0)
\]
Inequality path constraints:

\[ \text{Path constraints are formulated such that the aircraft have to follow the ILS glide path once the FAF has been passed} \]
Multi-Aircraft Optimization Problem

Inequality path constraints:

⇒ A certain separation distance between the aircraft has to be maintained

Aircraft distances:

\[
d_{ij}(t) = \sqrt{[x_i(t) - x_j(t)]^2 + [y_i(t) - y_j(t)]^2 + [z_i(t) - z_j(t)]^2}, \quad i = 1, ..., N, j = i + 1, ..., N
\]

Normalization of flight times w.r.t. final flight times:

\[
\tau_i = \frac{t_i}{t_{f,i}}, i = 1, ..., N
\]

Introduction of one single parameter for all flight times:

\[
t_f = t_{f,1} = t_{f,2} = ... = t_{f,N}
\]

⇒ The time elapsed is the same for all aircraft

⇒ Constraints w.r.t. the minimum distances can be checked directly (because of the direct correlation of the time elapsed)
Multi-Aircraft Optimization Problem

Cost function:

**Fuel-minimal approaches**: maximize aircraft masses at the final times

\[ J = -\sum_{i=1}^{N} m_i(t_{f,i}) \]

**Noise-minimal approaches**: minimize maximum sound pressure level or number of awakenings

**Integral cost functions**: 
- The same flight time for all aircraft is enforced
- Aircraft are located on different positions on the ILS glide path (i.e. they have covered different distances)
  - **Equal weighting** of the aircraft has to be achieved

Portion of the integral cost function originating from flight along ILS glide path is not incorporated into the integral cost function:

\[ \dot{m}_{\text{fuel,eff}}(t) = \dot{m}_{\text{fuel}}(t) \cdot 0.5 \cdot \left[ 1 - \tanh(a \cdot [x(t) - \Delta x_{\text{fuel}}]) \right] \]
Multi-Aircraft Optimization Problem

Full-Discretization Method – Forward (explicit) Euler:

⇒ Time discretization (e.g. equidistant):

\[ \tau_i = t_0 + (i - 1) \cdot h, \quad i = 1, \ldots, N, \quad h = \frac{t_f - t_0}{N - 1} \]

⇒ Discretization of controls and states at time discretization points:

\[ x_i, u_i, \quad i = 1, \ldots, N \]

⇒ Approximation of differential equations:

\[ x_{i+1} = x_i + h \cdot f(x_i, u_i, p), \quad i = 1, \ldots, N - 1 \]

⇒ Additional equality constraints:

\[ x_{i+1} - x_i - h \cdot f(x_i, u_i, p) = 0, \quad i = 1, \ldots, N - 1 \]
Multi-Aircraft Optimization Problem

Discretized Optimal Control Problem (Euler):

Determine the optimal parameter vector \( z = [x, u]^T \)

that minimizes the cost function \( J(z) \)

subject to

\( \Rightarrow \) the inequality constraints

\( C_{ineq}(x(z), z) \leq 0 \)

\( \Rightarrow \) and the equality constraints

\( C = \begin{pmatrix}
\psi_0(x(z), z) \\
C_{eq}(x(z), z) \\
x_{i+1} - x_i - h \cdot f(x_i, u_i) \\
r(x(z), z) \\
\psi_f(x(z), z)
\end{pmatrix} = 0 \)

\( \Rightarrow \) SNOPT (sequential quadratic programming SQP)
Multi-Aircraft Optimization Problem

Solution Strategy:

(1) Optimization without distance path constraints

(2) Simultaneous optimization with distance path constraints, using previous results as initial guess

⇒ Distance path constraints fulfilled by initial guess: Initial guess = Optimal solution of constrained problem

⇒ Distance path constraints not fulfilled by initial guess: Separation of aircraft until path constraints are met

Assumptions:

⇒ Optimal solution of unconstrained problem = Excellent initial guess of constrained problem

⇒ Cost function of a specific optimization problem is always less than or equal to the cost function of the same trajectory optimization problem with additional constraints
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Results

Generic scenario:

⇒ The optimal landing sequence and the optimal approach trajectories for four aircraft are sought

⇒ Initial conditions:

<table>
<thead>
<tr>
<th>AC No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>$x_{i,0}$</td>
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<td>−40000 m</td>
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<td>$y_{i,0}$</td>
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<td>6000 m</td>
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<tr>
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<td>−45.0°</td>
<td>95.0°</td>
<td>45.0°</td>
</tr>
<tr>
<td>$\gamma_{K,i,0}$</td>
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<td>0.0°</td>
<td>0.0°</td>
<td>0.0°</td>
</tr>
<tr>
<td>$V_{K,i,0}$</td>
<td>450.0 km/h</td>
<td>360.0 km/h</td>
<td>540.0 km/h</td>
<td>600.0 km/h</td>
</tr>
</tbody>
</table>
Results

Optimized approach trajectories

Optimized aircraft velocities $V_K$
Results

Optimized time histories of aircraft controls

Distances between aircraft

Optimal Scheduling of Fuel-Minimal Approach Trajectories
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Summary

⇒ The approach trajectories of multiple aircraft in the vicinity of an airport have been optimized **simultaneously**

⇒ Path constraints have been introduced so that the aircraft are finally located on the ILS glide path and keep a certain separation distance

⇒ The optimal landing sequence is determined by the optimization procedure

Outlook

⇒ Utilization of splines to describe the centerlines of the allowed flight path corridors for the involved aircraft

⇒ Introduction of path constraints w.r.t. the maximum lateral and horizontal deviation from the centerlines

⇒ More sophisticated distance path constraints between aircraft
Thank you very much for your attention!